

# Rayleigh - Taylor Gravity Waves and Quasiperiodic Oscillation Phenomenon in X-ray Binaries

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## ABSTRACT

Accretion onto compact objects in X-ray binaries (black hole, neutron star (NS), white dwarf) is characterized by non-uniform flow density profiles. Such an effect of heterogeneity in presence of gravitational forces and pressure gradients exhibits Rayleigh-Taylor gravity waves (RTGW). They should be seen as quasiperiodic wave oscillations (QPO) of the accretion flow in the transition (boundary) layer between the Keplerian disk and the central object. In this paper I show that the main QPO frequency, which is very close to the Keplerian frequency, is split into separate frequencies (hybrid and low branch) under the influence of the gravitational forces in the rotational frame of reference. The RTGWs must be present and the related QPOs should be detected in any system where the gravity, buoyancy and Coriolis force effects cannot be excluded (even in the Earth and solar environments). The observed low and high QPO frequencies are an intrinsic signature of the RTGW. I elaborate the conditions for the density profile when the RTGW oscillations are stable. A comparison of the inferred QPO frequencies with QPO observations is presented. I find that hectohertz frequencies detected from NS binaries can be identified as the RTGW low branch frequencies. I also predict that an observer can see the double NS spin frequency during the NS long (super) burst events when the pressure gradients and buoyant forces are suppressed. The Coriolis force is the only force which acts in the rotational frame of reference and its presence causes perfect coherent pulsations with a frequency twice of the NS spin. The QPO observations of neutron binaries have established that the high QPO frequencies do not go beyond of the certain upper limit. I explain this observational effect as a result of the density profile inversions. Also I demonstrate that a particular problem of the gravity waves in the rotational frame of reference in the approximation of very small pressure gradients is reduced to the problem of the classical oscillator in

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the rotational frame of reference which was previously introduced and applied for the interpretation of kHz QPO observation by Osherovich & Titarchuk.

*Subject headings:* Accretion, accretion disks —(magnetohydrodynamics:) MHD— stars:oscillations (including pulsations)— stars: neutron—X-ray:binaries

## 1. Introduction

The theory of oscillations of rotating fluids of variable density [Rayleigh- Taylor (R-T) effect] was developed in detail by Chandrasekhar (1961), hereafter C61. A large variety of the magnetohydrodynamic (MHD) problems, including the stability of inviscid and viscous fluids in the case of two uniform layers separated by a horizontal boundary with and without rotation as well as the magnetic field effects were analyzed using perturbation technique. The simplest case of a one-dimensional gravitational force was studied. A similar analysis was also implemented for the case of an exponentially varying density. It follows from C61 that the quasiperiodic oscillations (QPO) with the twin “kilohertz” frequencies represent the main frequencies of the stable gravity waves in the rotational frame of reference. It also as follows from C61 that the twin “kilohertz” frequencies should be on the order of the Keplerian frequency.

In this *Paper* I present the results of the study of the R-T effect for a particular case of fluid oscillations in an accretion flow under influence of a central gravitational force. This particular R-T analysis is important in view of the high and low frequency detection by the Rossi X-ray Timing Explorer (RXTE) in a number of low mass X-ray binaries (Strohmayer et al. 1996, van der Klis et al. 1996), black hole candidate sources (Morgan, Remillard & Greiner 1997; Strohmayer 2001a,b; Remillard 2002) and by Extreme Ultraviolet Explorer, Chandra X-ray observatory and optical observations in white dwarfs (Mauche 2002 and Woudt & Warner 2002). The presence of two observed peaks with frequencies  $\nu_1$  and  $\nu_2$  in the upper part of the power spectrum became a natural starting point in modeling the phenomena. In NS binaries, for example Sco X-1, the lower frequency part of the power spectrum, contains two horizontal branch oscillation (HBO) frequencies  $\nu_{HBO} \sim 45$  Hz and  $\nu_{2HBO} \sim 90$  Hz (probably the second harmonic of  $\nu_{HBO}$ ) which slowly increase with the increase of  $\nu_1$  and  $\nu_2$  (van der Klis et al. 2000). *Any plausible model faces the challenging task of describing the dependences of the peak separation  $\Delta\nu = \nu_2 - \nu_1$  on  $\nu_1$  and  $\nu_2$ .* Attempts have been made to relate  $\nu_1$  and  $\nu_2$  and the peak separation  $\Delta\nu = \nu_2 - \nu_1$  with the neutron star (NS) spin. In the sonic point beat frequency model by Miller, Lamb & Psaltis (1998) the kHz peak separation  $\Delta\nu$  is considered to be close to the NS spin frequency and thus  $\Delta\nu$  is predicted to be constant. However observations of kHz QPOs in a number

of binaries (Sco X-1, 4U 1728-34, 4U 1608-52, 4U 1702-429 and etc) show that the peak separation decreases systematically when the high [kilohertz (kHz)] frequencies increase (for a recent review see van der Klis 2000, hereafter VDK). For Sco X-1 VDK found that the peak separation of kHz QPO frequencies changes from 320 Hz to 220 Hz when the lower kHz peak  $\nu_1$  changes from 500 Hz to 850 Hz.

The correlation between high frequency (lower kHz frequency) and low frequency (broad noise component) QPOs previously found by Psaltis, Belloni & van der Klis (1999) for black hole (BH) and neutron star (NS) systems has been recently extended over two orders of magnitude by Mauche (2002) to white dwarf (WD) binaries. *Accepting the reasonable assumption that the same mechanism produces the QPO in WD, NS and BH binaries, one can argue that the data exclude relativistic models and, beat frequency models as well as any model requiring either the presence or absence of a stellar surface or a strong magnetic field.*

The transition layer model (TLM) was introduced by Titarchuk, Lapidus & Muslimov (1998), hereafter TLM98, to explain the dynamical adjustment of a Keplerian disk to the innermost sub-Keplerian boundary conditions (it is at the star surface for NSs and WDs). TLM98 argued that a shock should occur where the Keplerian disk adjusts to the sub-Keplerian flow. Thus the transition layer bounded between the sub-Keplerian boundary and the adjustment radius can undergo various type of oscillations under the influence of the gas, radiation, magnetic pressure and gravitational force. Osherovich & Titarchuk (1999), hereafter OT99, suggested that the phenomenological model of a one-dimensional classical oscillator in the rotational frame of reference could explain the observed correlations between twin kHz frequencies  $\nu_1$ ,  $\nu_2$  and the HBO frequencies. They further suggested that the oscillations of the fluid element that bounced from the disk shock region (at the adjustment radius) would be seen as two independent oscillations parallel and perpendicular to the disk plane respectively. This is due to the presence of a Coriolis force in the magnetospheric rotational frame of reference. In this paper I show that when the pressure gradients can be neglected, the problem of the Rayleigh-Taylor wave oscillations (buoyancy effect) in the rotational frame of reference is reduced to the OT99 formulation (see §3). This result provides a solid basis for application of the OT99 model for interpretation of the QPO phenomena observed in X-ray binaries.

The main goal of this *Paper* is to demonstrate that there is an inevitable effect of the gravity wave oscillations in the heterogeneous fluid of the accretion flow near compact objects. In §2, I formulate the problem of the gravity wave propagation in the bounded medium in the rotational frame of reference. In §3 I present analysis and solutions of this problem using a perturbation method in the context of three dimensional periodic waves with assigned wave numbers. Applications of the gravity modes and their relations with the

QPO observations is presented in §4. Summary and conclusions are drawn in §5.

## 2. The Problem of Gravity Wave Propagation in a Bounded Medium

Lord Rayleigh (1883) has treated the non-rotating inviscid case of the present problem. He developed a general theory for any density configuration  $\rho_0(z)$  (the  $z$  axis being the upward drawn vertical). Rayleigh's treatment of inviscid superposed fluids was extended by Bjerkness et al. (1933) to include the influence of rotation. With an assumption of the gravitational force directed vertically, Hide (1956) developed the theory for any density  $\rho_0(z)$  and viscosity  $\mu_0(z)$  profiles and any  $\delta$  (where  $\delta$  is an angle between the rotational axis and the vertical). In this study we consider a case of an inviscid and incompressible fluid.

The equation of relative motion appropriate to the problem is

$$\rho \frac{\partial u_i}{\partial t} + \rho u_i \frac{\partial}{\partial x_j} u_i - 2\rho \Omega \epsilon_{ijk} u_j s_k = -\frac{\partial p}{\partial x_j} - g \rho e_i, \quad (1)$$

where the fluid is supposed to rotate uniformly about an axis whose direction is specified by the unit vector  $\mathbf{s} = \mathbf{\Omega}/\Omega$ . The tensor notation follows the summation convention and the unit vector  $\mathbf{e}$  in the radial direction is introduced. In equation (1)  $\rho$  denotes the density,  $\Omega$  the angular velocity of rotation,  $u_i$  the  $i$ th component of the (Eulerian) velocity vector,  $p$  is the pressure and  $g$  is the acceleration due to gravity. This term  $g$  can represent the effective gravity which includes the centrifugal and radiation pressure forces. For an incompressible fluid the continuity equation is

$$\frac{\partial u_i}{\partial x_i} = 0. \quad (2)$$

Because (similar to C61) diffusion effects are ignored in this analysis, an individual fluid element retains the same density throughout its motion. Hence it is required that

$$\frac{D\rho}{Dt} = \frac{\partial \rho}{\partial t} + u_j \frac{\partial \rho}{\partial x_j} = 0. \quad (3)$$

Because the equilibrium situation in the comoving frame is a static one it is characterized by  $u_i = 0$ . We now assume the equilibrium situation to be slightly disturbed, so that  $u_i \neq 0$ . So we shall write

$$\rho = \rho_0(r, z) + \delta\rho(x, y, z), \quad p = p_0(z) + \delta p(x, y, z, t). \quad (4)$$

and treat  $u_i$ ,  $\delta\rho$ , and  $\delta p$  as quantities of the first order of smallness so that products of such quantities can be ignored.

We are interested in the study of the gravity wave oscillations in the disk transition layer where the accretion flow is sub-Keplerian and the temperature of the flow is of order 5 keV or more (see cartoon diagram of the system in Titarchuk, Osherovich & Kuznetsov 1999, hereafter TOK, Fig. 1). These temperatures of order of 5 keV are a representative values for plasma temperatures inferred from X-ray spectra during kHz QPO events (see more in Titarchuk, Bradshaw & Wood 2001 and Titarchuk & Wood 2002).

At this stage we introduce the Cartesian coordinate system. We take the  $z$  axis to be in the direction of the (upward) vertical and the  $x$  axis to be such that the  $(x, z)$  plane contains the angular velocity vector  $\boldsymbol{\Omega}$  which then has components  $(\Omega_x, 0, \Omega_z)$ . In the general case the gravitational force has components  $g(x/R, y/R, z/R)$  where  $R = (x^2 + y^2 + z^2)^{1/2}$ .

If  $(u, v, w)$  are the components of the velocity  $\mathbf{u}$ , then on substituting in equations (1), (2) and (3) we find

$$\rho_0 \frac{\partial u}{\partial t} - 2\rho_0 \Omega_z v = -\frac{\delta p}{\delta x} - g_x \delta \rho, \quad (5)$$

$$\rho_0 \frac{\partial v}{\partial t} - 2\rho_0 \Omega_x w + 2\rho_0 \Omega_z u = -\frac{\delta p}{\delta y} - g_y \delta \rho, \quad (6)$$

$$\rho_0 \frac{\partial w}{\partial t} + 2\rho_0 \Omega_x v = -\frac{\delta p}{\delta z} - g_z \delta \rho, \quad (7)$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0, \quad (8)$$

$$\frac{\partial \delta \rho}{\partial t} + u \frac{\partial \rho_0}{\partial x} + v \frac{\partial \rho_0}{\partial y} + w \frac{\partial \rho_0}{\partial z} = 0. \quad (9)$$

One can see that the variation of  $\rho$  is ignored in all terms except for the ones representing the buoyancy force (see C61 for details).

### 3. Analysis of various types of the solutions of the problem

In order to illustrate the buoyancy (R-T) effect we consider the simplest solutions of the problem including those which already exist in the literature.

#### Case A

In the case when we allow the vector  $\boldsymbol{\Omega}$  to rotate around the  $z$  axis at angle  $\delta$ , we introduce the cylindrical coordinate system, namely taking the  $z$  axis in the direction of the (upward) vertical, the  $x$  axis as the radial axis and the  $y$  axis as the azimuthal axis in the horizontal plane. Then the vector  $\boldsymbol{\Omega}$  has components  $(\Omega \sin \delta, 0, \Omega \cos \delta)$  and gravitational force vector has components  $g(r/R, 0, z/R)$  where  $r = (x^2 + y^2)^{1/2}$ .

(I) With assumptions that  $\rho_0$  is only a function of  $R$  the scale height of  $\rho_0$  is order of  $R$  we obtain that  $\delta\rho = [(d\rho/dx)dx + (d\rho/dz)dz] \sim \rho_0/R[(r/R)\chi + (z/R)\zeta]$  where  $\chi = dx$ ,  $\zeta = dz$  are the radial and vertical components of displacement respectively and consequently the radial and vertical components of the perturbation velocity are  $u = d\chi/dt$ ,  $w = d\zeta/dt$ . Thus the vector  $\mathcal{G} = \delta\rho(g_x, 0, g_z)$  is transformed into  $\mathcal{G} = \rho_0(g/R)[(r/R)(r\chi/R + z\zeta/R), 0, (z/R)(r\chi/R + z\zeta/R)]$ .

Furthermore if we neglect the effects of pressure gradients, the set of equations (5-7) can be reduced to the system where the right hand side consists of only the vector  $\mathcal{G}$ . Then, for radial, azimuthal and vertical displacements  $\chi$ ,  $\Upsilon$ ,  $\zeta$  this system takes the form:

$$\ddot{\chi} - 2\Omega \cos \delta \dot{\Upsilon} = -(g/R)(r/R)(r\chi/R + z\zeta/R), \quad (10)$$

$$\ddot{\Upsilon} + 2\Omega \cos \delta \dot{\chi} - 2\Omega \sin \delta \dot{\zeta} = 0, \quad (11)$$

$$\ddot{\zeta} + 2\Omega \sin \delta \dot{\Upsilon} = -(g/R)(z/R)(r\chi/R + z\zeta/R). \quad (12)$$

OT99 have already analyzed the solution of Eqs. (10-12) (see Eqs. 2-4, in OT99) in the case of  $z/R \ll 0$  and  $\delta \ll 1$ . They found that in the rotational frame of reference, the radial oscillation with the main frequency  $\omega_K = (g/R)^{1/2}$  is split to the oscillations taking place near the horizontal (disk) plane ( $\chi\Upsilon$ ) with the hybrid frequency  $\omega_h = (\omega_K^2 + 4\Omega^2)^{1/2}$  and to oscillations taking place near the vertical plane ( $\Upsilon\zeta$ ) with the low branch frequency  $\omega_L = 2\Omega(\omega_K/\omega_h) \sin \delta$ .

For  $z \sim r$  the dispersion equation for the frequency  $\omega$

$$\omega^2[\omega^4 - 2(\omega_*^2 + 2\omega^2)\omega^2 + \omega_*^4 + 4\Omega^2\omega_*^2 + 2\Omega^2\omega_*^2 \sin 2\delta] = 0 \quad (13)$$

besides the nonoscillatory mode ( $\omega = 0$ ), describes two oscillatory eigenmodes. For  $\delta \ll 1$  they are  $\omega_1 = \omega_* = (g/2R)^{1/2} = \omega_K/\sqrt{2}$  and  $\omega_2 = (\omega_*^2 + 4\Omega^2)^{1/2} = (\omega_K^2/2 + 4\Omega^2)^{1/2}$ .

The relation of the model eigenfrequencies  $\nu_L = \omega_L/2\pi$ ,  $\nu_K = \omega_K/2\pi$  and  $\nu_h = \omega_h/2\pi$  with the QPO observed frequencies: horizontal branch oscillation frequencies  $\nu_{HBO}$ , kHz frequencies were studied by OT99; TOK, Kuznetsov & Titarchuk (2002); and Titarchuk (2002), hereafter T02. It is worth noting that the relation between  $\nu_K$ ,  $\Omega$ ,  $\nu_h$  and  $\nu_L$  predicts the existence of the invariant quantity  $\delta$ . (e.g. Titarchuk & Osherovich 2001). T02 calculated  $\delta$  and its uncertainty of  $\delta$  finding that the inferred  $\delta$ -values are consistent with being constant at least for four Z sources, Sco X-1, GX 340+0, GX 5-1, GX 17+2 (see more on this issue in section §4).

(II). With assumptions that  $g\Delta\rho \ll 1$  and  $|\nabla p| \ll 1$  the set of equations for  $\chi$ ,  $\Upsilon$ ,  $\zeta$  is similar to Eqs (10-12) where the right hand side vector is the zero vector,  $\mathbf{0} = (0, 0, 0)$ . In this case the dispersion equation for  $\omega$  ( for small  $\delta \ll 1$ )

$$\omega^2(\omega^2 - 4\Omega^2) = 0 \quad (14)$$

has the only one nontrivial root  $\omega = 2\Omega$  which is related to the eigenmode oscillations taking place parallel to the disk ( $\chi\Upsilon$ )-plane. We discuss the application of this solution to the observation in section §4.

### Case B

If we do not neglect the pressure gradient and assume the vertical gravitational force and the vector  $\Omega$  directed along the vertical ( $\delta = 0^0$ ), then the entire problem (Eqs. 5-9) is reduced to set of equations which have already been analyzed by Chandrasekhar in C61. He studied two cases: (1) In the case of two uniform fluids separated by a horizontal boundary he showed that for two adjacent, hydrostatic, inviscid fluids, the low fluid having density  $\rho_1$  and the upper layer having density  $\rho_2$  the eigenfrequency  $\nu_{R-T}$  was

$$\nu_{R-T} = [2(\Omega/2\pi)^2 + (4(\Omega/2\pi)^4 + \nu_0^4)^{1/2}]^{1/2} \quad (15)$$

where  $\nu_0$  is the frequency in the absence of rotation. The frequency  $\nu_0$  is of order of  $\nu_K = (g/R)^{1/2}$  if the k-wave number ( $\nu_0$  depends on  $k$  and density difference  $\Delta\rho = \rho_2 - \rho_1$ ) of order  $R^{-1}$  and  $\Delta\rho/(\rho_1 + \rho_2) \sim (0.5 - 1)$ .

For a stratified medium of density  $\rho = C_0 \exp(\beta z)$ , where  $1/\beta$  is the scale height, the R-T instability occurs for positive  $\beta$ , whereas stable gravity waves occur for negative  $\beta$ . The R-T frequency  $\nu_{R-T} = \nu_h = [\nu_0^2 + 4(\Omega/2\pi)^2]^{1/2}$  if one assumes that the wave number  $k$  is of order of  $d^{-1} \sim H^{-1}$  (where  $d$  is a layer size). It is worthwhile to emphasize that even in the simplest case of the vertical gravitational force (C61) the hybrid frequency  $\nu_h$  is an eigenfrequency (which is also true for the case with no pressure gradient effects, see above). Formula (10) for  $\nu_{R-T}$  can also be reduced to  $\nu_h$  if one assumes that  $\nu_0 \sim 2(\Omega/2\pi)$ .

### Case C

Now we consider the case when the vector  $\Omega$  rotates uniformly around the vertical at angle  $\delta$ , then the gravitational vector  $\mathcal{G} = \delta\rho(gr/R, 0, zg/R)$ , also taking into account the pressure effects. The general case of  $\mathcal{G} = \delta\rho(g_x, g_y, g_z)$  can be analyzed in a similar way and it will be presented elsewhere. Following the usual practice in problems of this kind, we seek solutions of equations (5-9) which are of form (see e.g. C61)

$$u, v, w, \delta\rho, \delta p = \text{constant} \times \exp(ik_x x + ik_y y + ik_z z + nt), \quad (16)$$

where  $k_x, k_y$  and  $k_z$  are the horizontal and vertical wave numbers of the harmonic perturbations respectively. We also assume that  $\rho_0 = f(x)\varphi(z)$  is a function of  $x$  (or  $r$ ) and  $z$  which the scale heights are  $1/\gamma$  and  $1/\beta$  for radial and vertical density profiles respectively,  $r \approx R$ . Upon substituting for  $u, v, w, \delta\rho, \delta p$  in the form (16) equations (5) to (8) become

$$nu - 2\Omega_z v = -ik_x(\delta p/\rho_0) + gu\gamma/n + gw\beta/n, \quad (17)$$

$$nv - 2\Omega_x w + 2\Omega_z u = -ik_y(\delta p/\rho_0), \quad (18)$$

$$nw - 2\Omega_x v = -ik_z(\delta p/\rho_0) + (zg/R)u\gamma/n + (zg/R)w\beta/n, \quad (19)$$

$$k_x u + k_y v + k_z w = 0. \quad (20)$$

We also use the relation  $n\delta\rho + (\gamma u + \beta w)\rho_0 = 0$  which follows from Eqs. (9) and (16) in order to express  $\delta\rho$  through  $\rho_0$ ,  $u$  and  $w$ . The set of equation (17-20) assumes a nontrivial solution only if the determinant of the system  $\mathcal{D} = 0$ . This equation provides the dispersion relation for the determination of  $n$ :

$$P(n) = a_4 n^4 + a_2 n^2 + a_1 n + a_0 = 0, \quad (21)$$

where  $a_0 = k_y^2 g^2 \gamma^2 (k_z/k_x - z/R)(\beta/\gamma - k_z/k_x)$ ,  $a_1 = -\beta k_y g (2\Omega_x k_x + 2\Omega_z k_z)(1 - \gamma z/\beta R)$ ,  $a_2 = 4(\Omega_x k_x + \Omega_z k_z)^2 - g\beta[-(\gamma/\beta)k_x^2(\beta/\gamma - k_z/k_x)(k_z/k_x - z/R) + k_y^2(\gamma/\beta + z/R)]$ ,  $a_4 = k^2 = k_x^2 + k_y^2 + k_z^2$ .

### 3.1. Stable gravity modes

The specific wave values  $k_x, k_y, k_z$  are determined by the conditions imposed on the oscillatory domain boundary. Thus for a given set of boundary conditions the analysis of the R-T instability is reduced to the analysis of the roots of fourth order algebraic equation (21), which depends on the main parameters of the atmosphere,  $\beta$ ,  $\gamma$  and ratio of the wave numbers  $k_z/k_x$ . We assume that  $z/R < 1$  and  $k_x, k_y \propto 1/R$ ,  $k_z \propto 1/z$  is the case of interest.

Figure 1 illustrates the specific behavior of the polynomial  $P(n)$  which should help one to understand the presence (or absence) of its roots for given coefficient  $a_0, a_1, a_2$  ( $a_4 > 0$ ,  $a_3 = 0$ ). For example, if  $a_0 < 0$ ,  $a_1 < 0$  and  $a_2 > 0$  (see case *i* below) then  $P(n)$  has two complex conjugate roots  $n_{1,2} = \zeta \pm i\eta$  and two real roots  $n_3, n_4$  (in fact,  $d^2 P/dn^2 > 0$ ). Because  $a_1 < 0$ , the absolute value of the positive root  $n_4$  is larger than that of the negative one  $n_3$ . But  $a_3$  is a sum of the polynomial (real and complex conjugate) roots, and because of  $a_3 = n_1 + n_2 + n_3 + n_4 = 0$  one can come to conclusion that  $2\zeta = -(n_3 + n_4) < 0$ . Therefore (in this case) the stable oscillatory mode exists and  $n_{1,2}$  are related to these damped oscillations.

Below we present all cases with the oscillatory stable solutions:

*Case i:*  $\beta, \beta/\gamma > 0$ ,  $\beta/\gamma - k_z/k_x < 0$  and  $a_2 > 0$  (see Fig. 1, curve i). For such conditions  $a_0 < 0$  and  $a_1 < 0$ . Then equation (21) has two real roots which relate to the unstable (growing) and stable (decaying) modes with one pair of complex conjugate roots corresponding to the stable oscillatory mode:

$$n_{1,2} \approx a_1/2d \pm i\omega_h, \quad (22)$$



where  $\omega_h = [(|a_2| + d)/2a_4]^{1/2}$  and  $d = (a_2^2 - 4a_4a_0)^{1/2}$ . These roots of equation (21) are found using the sequential approximation method: first we solve equation (21) with  $a_1 = 0$ ,  $\tilde{n}$  and then in the next stage we look for roots of equation (21) as  $n = \tilde{n} + \alpha$ .

*Case ia:*  $\beta, \beta/\gamma > 0, \beta/\gamma - k_z/k_x < 0$  and  $a_2 < 0$ . For such conditions  $a_0 < 0$  and  $a_1 < 0$ . Equation (21) has just one pair of the complex conjugate roots and two real roots which relate to the unstable (growing) and stable (decaying) modes (see Fig. 1, curve ia). The oscillatory mode  $n_{1,2}$  is stable for which we have

$$n_{1,2} \approx a_1/2d \pm i\omega_L, \quad (23)$$

where  $\omega_L = [(-|a_2| + d)/2a_4]^{1/2}$ .

*Case ib:*  $\beta > 0, \beta/\gamma < 0, \beta/\gamma - k_z/k_x < 0$ . For such conditions  $a_0 < 0$  and  $a_1 < 0$  and  $a_2 > 0$ . This case is similar to case i (see Fig 1, curve i) when equation (21) has two real roots which relate to the unstable (growing) and stable (decaying) modes and complex conjugate roots which correspond to the stable oscillatory mode:

$$n_{1,2} \approx a_1/2d \pm i\omega_h. \quad (24)$$

*Case ii:*  $\beta > 0, \beta/\gamma - k_z/k_x > 0$ . For such conditions  $a_0 > 0$  and  $a_1 < 0$  and  $a_2 > 0$ . Equation (21) has a pair of complex conjugate roots (see Fig. 1, curve ii) and one of them  $n_{1,2}$  is related to the stable oscillatory mode:

$$n_{1,2} \approx a_1/2d \pm i\omega_h. \quad (25)$$

*Case iii:*  $\beta < 0, \beta/\gamma - k_z/k_x > 0$ , and  $a_2, d > 0$ . For such conditions  $a_0 > 0$  and  $a_1 > 0$ . Equation (21) has a pair of complex conjugate roots (see Fig. 1, curve iii) and one of them  $n_{3,4}$  is related to the stable oscillatory mode:

$$n_{1,2} \approx -a_1/2d \pm i\omega_L, \quad (26)$$

where  $\omega_L = [(a_2 - d)/2a_4]^{1/2}$ .

Finally, we single out the case when the effective gravitational force goes to zero. This can happen during a burst event, when the gravitational forces are compensated by the radiation pressure forces (e.g. Titarchuk 1994). For such conditions,  $a_0, a_1 \rightarrow 0$  and  $a_2/a_4 = 4[(k_x/k)\Omega_x/k + (k_z/k)\Omega_z]^2$ . Equation (21) has zero roots  $n = 0$  and conjugate complex roots

$$n_{1,2} = \pm i(a_2/a_4)^{1/2} = \pm 2i[(k_x/k)\Omega_x + (k_z/k)\Omega_z] \quad (27)$$

which are related to a pure harmonic mode. In fact, this result also follows from treatments detailed in C61 and OT99. The hybrid frequency then becomes simply  $\nu_h = 2(\Omega/2\pi)$  for

$g = 0$ . This case of the perfect coherent oscillations would also occur if the density profile is quasi-uniform i.e. when  $\beta, \gamma \rightarrow 0$  but  $\beta/\gamma = O(1)$ . These two conditions (on either the effective gravity force or the density profile) can be realized during the burst event.

In this section the main goal is to reveal all cases when stable gravity modes exist and when the stability breaks down. This analysis is particularly important in a view of the transient nature of QPO features (see e.g. Zhang et al. 1998). With an increase of bolometric luminosity (presumably in mass accretion rate) the kHz QPO frequency increases and then entirely disappears! At low rates it appears once again! The stability analysis presented here leads to conditions for the existence and the destruction of gravity modes in terms of density profiles scale heights  $\beta^{-1}$ ,  $\gamma^{-1}$  and boundary conditions  $(k_x, k_y, k_z)$ . In fact, the strong dependence of the gravity wave stability on the density profile was a central point of Chandrasekhar's analysis (C61). For the accretion disk cases, we can give an example where stable gravity modes are followed by instabilities (the gravity mode destruction). In case (ib) we have a stable mode with  $\omega_h$  as a QPO frequency. When the density profile changes, over  $z$ -coordinate from  $\gamma < 0$  to  $\gamma > 0$  then the stable  $g$ -mode with  $\omega_h$  can still be sustained [see case (ia)], But if the density profile over  $z$  coordinate is stabilized ( $\beta < 0$ ) and that over  $r$  coordinate is inverted (i.e. from  $\gamma < 0$  to  $\gamma > 0$ ) then the QPO oscillations are no longer stable.

#### 4. Gravity modes and their relation to QPO phenomenon

As we have seen in the previous section, there are two stable oscillatory gravity modes: one is associated with the hybrid frequency  $\omega_h$  (see cases *i*, *ib*, *ii*) and another with the low branch frequency  $\omega_L$  (cases *ia* and *iii*). We also estimate the decay rate for oscillations as  $\lambda = |a_1|/d$  and the QPO quality value  $Q = \omega/2\lambda$ . The presence and absence of these modes depend on the atmospheric structure (scale height inverses  $\beta$  and  $\gamma$ ) and on the imposed boundary conditions (wave numbers  $k_x, k_y$  and  $k_z$ ). Furthermore, it is possible to restore the related boundary conditions if one compares the observed QPO features with the calculated mode frequencies and  $Q$ -values. The analysis made in section 3 is also necessary in order to control an accuracy of numerical calculations of the set of hydrodynamical equations (5-9).

To illustrate the results obtained in §3 we should specify the orders of the introduced quantities  $k_x, k_y, k_z, \beta, \gamma$ . Namely we assume that  $k_x \sim 1/R$ ,  $k_y \sim 1/R$ ,  $k_z \sim 1/z$ ,  $\gamma \sim q/R$  (where  $|q| < 1$ ) and  $z/R < 1$ . If we also assume that  $\beta/\gamma - k_z/k_x \sim R/z$  then  $\beta \sim 2q/z$ . With these assumptions expressions for the polynomial coefficients  $a_j$  (see Eq. 21) are significantly simplified:  $a_0/a_4 \sim q^2\omega_K^4$ ,  $a_1/a_4 \sim -2|q|\omega_K^2(z\Omega_x/R + \Omega_z)$ ,  $a_2/a_4 \sim 4(z\Omega_x/R + \Omega_z)^2 + q\omega_K^2$ .

Because  $|q| < 1$  and  $\omega_K < \omega_h$  we can calculate  $\omega_L$  as

$$\omega_L \approx |q|(\omega_K/\omega_h)\omega_K \quad (28)$$

and

$$Q_L = \omega_L/2(a_1/2d) \approx \omega_h/2\Omega \geq 1/2. \quad (29)$$

TOK introduced the classification scheme for the QPO features and related the observed high and low kHz frequencies to the hybrid and Keplerian frequencies,  $\nu_h = \omega_h/2\pi$  and  $\nu_K = \omega_K/2\pi$  respectively. They also attributed the hectohertz frequencies detected in the atoll source 4U 1728-34 (Ford & van der Klis 1999) to the low branch. The angle  $\delta$  was found to be almost twice that found in Sco X-1 (see also T02 for details of  $\delta$ -determination). The low frequencies in atoll sources (as a rule) are three times higher than those in the Z-sources. The hectohertz frequencies have been identified in several other neutron star LMXBs (4U 0614+09; van Straaten et al. 2000, 2002; SAX J1808.4-3658, 4U 1705-44; Wijnands & van der Klis 1998). They weakly depend on the kHz frequencies, their ratio to low kHz frequency being about (4-5) (van Straaten et al. 2002, hereafter S02). The hectohertz frequency Lorentzian profile are very broad with  $Q$ -values around 1 or even less. We suggest that these observed frequencies can be identified as the low branch frequencies (see formulas 28-29). Taking into account resonance effects (T02), we correct the error bars of the hectohertz frequencies presented in S02. In Figure 2 the best fit to the data (S02) is presented using formula (28) which includes one fit parameter  $q$ . The best-fit value of  $|q| = 0.3$  for which  $\chi^2_{red} = 0.76$ . The estimated  $Q \gtrsim 1/2$  are also in agreement with the observed values of  $Q$  (S02, Table 2).

The harmonic modes  $\omega_{hm} = 2(k_x\Omega_x + k_z\Omega_z)$  can be related to a coherent oscillation frequency of 582 Hz which is observed in 4U 1636-53 during the superburst (Strohmayer & Markwardt 2002). If  $k_z/k_x \gg 1$  (which is our case)  $\omega_{hm}$  is a double frequency of the NS spin frequency. Furthermore, we have already found the same eigenmode with the frequency  $2\Omega$  in case (AII) when effects of the buoyant forces and pressure gradients are neglected (see §3). In fact, Miller (1999) has reported the detection of the coherent pulsations of frequency  $\sim 291$  Hz during the burst development in 4U 1636-53. *Thus the NS spin frequency  $\Omega/2\pi = 291$  Hz and the double NS spin frequency  $2\Omega/2\pi = 582$  Hz have probably been detected in 4U 1636-53.*

It is also worth noting that the RXTE observations of neutron binaries establish that the kHz QPO frequencies do not seem to exceed beyond a certain upper limit (Zhang et al. 1998). This observational effect may be a result of the density profile. Our stability analysis presented in §3 clearly indicates such a possibility (see also C61).

## 5. Conclusions

I have presented a detailed study of the Rayleigh-Taylor (R-T) instability in the accretion flow. To summarize I have : (1) put forth arguments to explain the QPO phenomena, as a result of the R-T effect in the rotational frame of reference. (2) formulated and solved the mathematical problem of the gravity wave propagation (R-T effect) in the accretion flow. (3) concluded that the stable gravity modes in the rotational frame of reference are related to the hybrid and low branch frequencies. (4) demonstrated that the particular problem of the gravity waves in the rotational frame of reference, in the approximation of very small pressure gradients, is reduced to the problem of the classical oscillator in the rotational frame of reference which was previously introduced and applied for the interpretation of kHz QPO observation by OT99.

I demonstrate that these frequencies are intrinsic features of the R-T effect. They appear in various configurations of the accretion flow depending on assumptions regarding the density profiles, the boundary conditions and the effects of the pressure forces. It is not by chance the high and low frequencies phenomenon has common observational appearances for a wide range of objects classes from black hole sources down to white dwarfs (Mauche 2002). (5) Investigated the conditions for the density profile and the wave numbers (boundary conditions) when the gravity modes are stable. (6) Identified the observed QPO frequencies seen in the power density spectra of NS LMXBs using the inferred gravity mode frequencies. In particular, I found that the inferred low branch frequencies and their Q-values are consistent with the QPO hectohertz frequencies observed in the atoll sources 4U 1728-34 and 4U 0614+19. (7) During the NS long (super) burst event, I find that the observer should see oscillations at double NS spin frequency. The Coriolis force is the only force which acts in the rotational frame of references and its presence causes perfect coherent pulsations with a frequency twice of the NS spin frequency.

Finally one can conclude that the R-T gravity wave oscillations must be present and the related QPOs should be detected in any system where the gravity, buoyancy and Coriolis force effects cannot be excluded (even in the Earth and solar environments).

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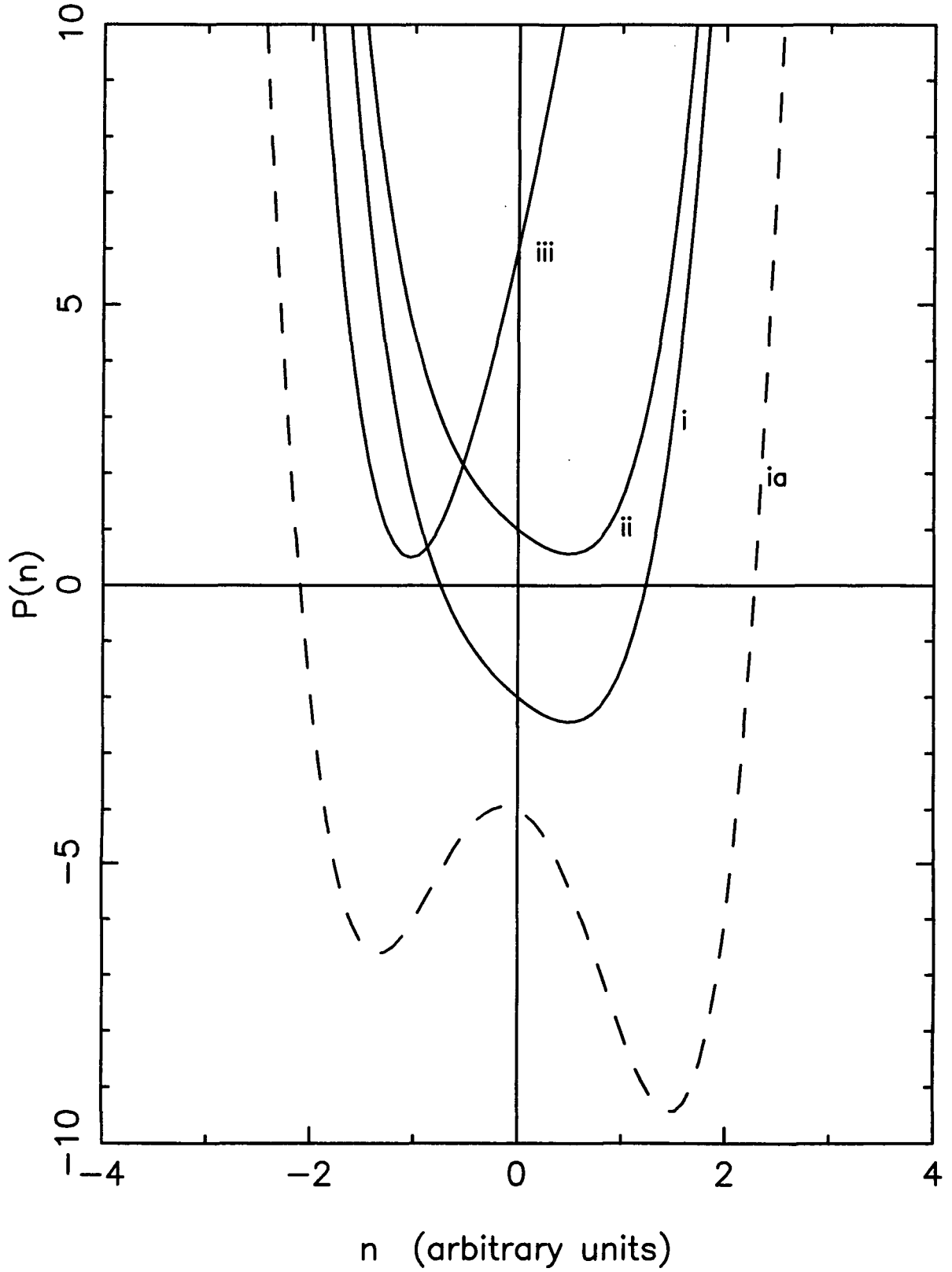


Fig. 1.— Behavior of the dispersion polynomial  $P(n)$  for stable gravity modes (see text).

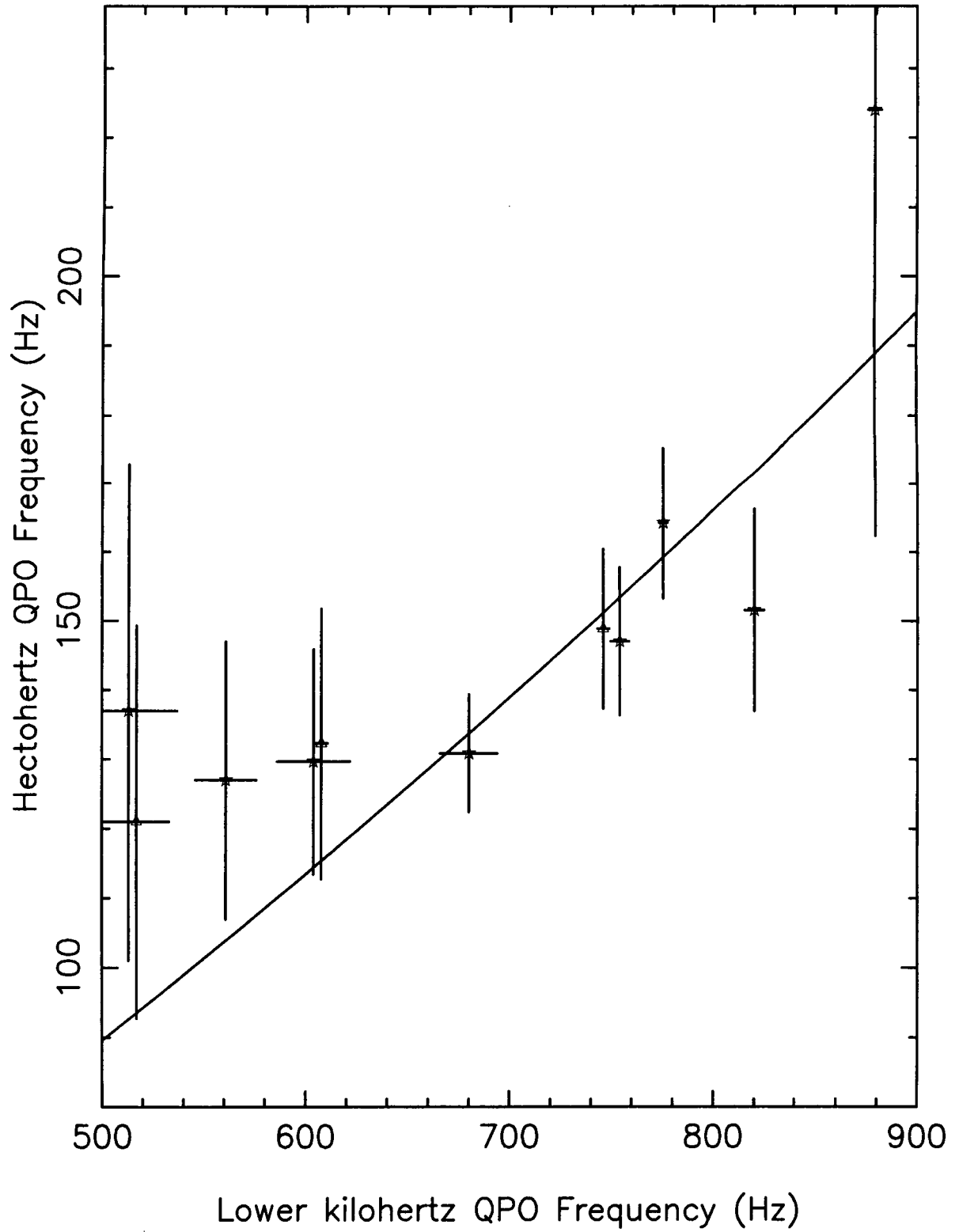


Fig. 2.— Correlation between the lower kHz frequency and hectohertz frequency for 4U 1728-34 (star), 4U 0614+09 (triangles) (van Straaten et al. 2002) and the best-fit curve of low branch frequency vs Keplerian frequency (solid line).  $\chi^2_{red} = 0.76$  for  $|q| = 0.3$ .